

Quantum anomalies at horizon and Hawking radiations in Myers-Perry black holes

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ABSTRACT: A new method has been developed recently to derive Hawking radiations from black holes based on considerations of gravitational and gauge anomalies at the horizon [1, 2]. In this paper, we apply the method to Myers-Perry black holes with multiple angular momenta in various dimensions by using the dimensional reduction technique adopted in the case of four-dimensional rotating black holes [3].

KEYWORDS: Space-Time Symmetries, Anomalies in Field and String Theories, Black Holes.

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1. Introduction

Hawking radiation is the quantum effect to arise for quantum fields in a background space-time with an event horizon. There are many different derivations, from the original calculation based on Bogoliubov transformations [4] to Euclidean approaches [5], and all of them universally give the same answer. The universality tells us that the Hawking radiation must be determined only by some universal quantum effects just on the horizon.¹

In the seminal paper by Robinson and Wilczek [1], it was proposed that the flux of Hawking radiation can be fixed by the amount of the gravitational anomaly at the horizon. The method was then generalized in [2] to charged black holes by using the gauge anomaly in addition to the gravitational anomaly and further applied to rotating black holes [3, 7] and others [8]. The essential observation in [1] is that quantum fields near the horizon behave as an infinite set of two-dimensional fields and ingoing modes at the horizon can be considered as left moving modes while outgoing modes as right moving modes. Once the ingoing modes fall into the black hole, they never come out classically and cannot affect the physics outside the black hole. Quantum mechanically, however, they cannot be neglected because, without the ingoing modes, the theory becomes chiral at the horizon, which makes the effective theory anomalous under general coordinate or gauge transformations. In this sense, the ingoing modes at the horizon only affect the exterior region through quantum anomalies. This is the basic idea of the method but there is a slight difference between the original calculation in [1] and that in [2]. In this paper, we adopt the calculation used in [2]. We will explain the difference in the appendix.

¹The effect of scatterings away from the horizon changes the spectrum to gray. But this is not the universal part of the Hawking radiation and we do not discuss it in the present paper.

This approach is similar to the beautiful derivation of the Hawking flux based on conformal anomalies [6], in which the Hawking flux was obtained by solving the conservation law of the energy-momentum tensor with the information of conformal anomaly. In our method, instead of conformal anomaly, we use gravitational or gauge anomalies. Both anomalies are quantum effects but there are the following differences. The gravitational or gauge anomalies arise only for chiral theories while conformal anomaly can arise even for vector-like theories. The reason that the gravitational and gauge anomalies are relevant to the Hawking radiation is due to the chiral decomposition property near the horizon. Namely, quantum fields near the horizon can be decomposed into the left (ingoing) and right (outgoing) modes and the left modes are causally decoupled from the exterior physics classically. Also the gravitational and gauge anomalies are independent of the details of quantum fields and can be universally determined. Another advantage to use the gauge anomalies is that we can derive the fluxes of charges and angular momenta in addition to the energy flux.

In this paper, we apply the method to higher-dimensional Myers-Perry (MP) black holes [9]. Hawking flux from MP black holes with a single charge (angular momentum) was obtained in [7] but they could not obtain the flux in cases with multiple angular momenta. In this paper, by using similar dimensional reduction technique adopted in [3], we show that the anomaly method can be similarly applied to MP black holes with more than one charge, and that it reproduces the flux of each angular momentum F_{ai} and that of energy-momentum tensor F_M associated with the Hawking radiation by each partial wave,

$$\begin{aligned}
 F_{ai} &= \int_0^\infty \frac{d\omega}{2\pi} m_i \left(N_{\{m_j\}}(\omega) - N_{\{-m_j\}}(\omega) \right) = \frac{m_i}{2\pi} \sum_{j=1}^n \frac{m_j a_j}{r_H^2 + a_j^2}, \\
 F_M &= \int_0^\infty \frac{d\omega}{2\pi} \omega \left(N_{\{m_j\}}(\omega) + N_{\{-m_j\}}(\omega) \right) = \frac{1}{4\pi} \left(\sum_{i=1}^n \frac{m_i a_i}{r_H^2 + a_i^2} \right)^2 + \frac{\pi}{12\beta^2}. \quad (1.1)
 \end{aligned}$$

Here $N_{\{m_i\}}(\omega)$ is the Planck distribution with an inverse-temperature β and chemical potentials $a_i/(r_H^2 + a_i^2)$ for the angular momenta m_i ($i = 1, 2, \dots, n$).² These fluxes are given by the sums of contributions from a particle with charge $\{m_i\}$ and antiparticle with charge $\{-m_i\}$.

The organization of the paper is as follows. In section 2, we consider quantum fields in the background of the MP black holes in various dimensions and show that they behave as an infinite set of two-dimensional conformal fields near the event horizon. In section 3, we consider symmetries for them and relate the conservation laws of the original energy-momentum tensor to those in the dimensionally reduced theories. In section 4, we obtain the Hawking fluxes based on two-dimensional gravitational and gauge anomalies. Section 5 is devoted to discussions. In appendix, we explain the difference between the original calculation in [1] and that in [2].

²Here we used the Planck distribution for fermions in order to avoid the problem of superradiance which is related to scatterings away from the horizon.

2. Quantum fields in Myers-Perry black hole

In the $D = 2n + 1 + \epsilon$ ($\epsilon = 0$ or 1) dimensions, the metric of the Myers-Perry black hole is given by [9]

$$ds^2 = dt^2 - \epsilon r^2 d\alpha^2 - \sum_{i=1}^n (r^2 + a_i^2) (d\mu_i^2 + \mu_i^2 d\phi_i^2) - \frac{\mu r^{2-\epsilon}}{\Pi F} \left(dt - \sum_{i=1}^n a_i \mu_i^2 d\phi_i \right)^2 - \frac{\Pi F}{\Pi - \mu r^{2-\epsilon}} dr^2, \quad (2.1)$$

where

$$F = 1 - \sum_{i=1}^n \frac{a_i^2 \mu_i^2}{r^2 + a_i^2}, \quad (2.2)$$

$$\Pi = \prod_{i=1}^n (r^2 + a_i^2). \quad (2.3)$$

The following constraint is imposed for μ_i ($i = 1, 2, \dots, n$) and α ,

$$\sum_{i=1}^n \mu_i^2 + \epsilon \alpha^2 = 1, \quad (0 \leq \mu_i \leq 1, \quad -1 \leq \alpha \leq 1). \quad (2.4)$$

This metric describes a black hole space-time with the mass $M = \frac{(D-2)A_{D-2}}{16\pi G} \mu$ and angular momenta $\frac{2}{D-2} M a_i$ in the ϕ_i -directions, where A_{D-2} is the volume of S^{D-2} . This black hole is stationary and has $U(1)^n$ isometries with the Killing vectors ∂_{ϕ_i} . We assume the existence of horizons located at positive solutions of $\Pi - \mu r^{2-\epsilon} = 0$. The inverse of metric is given by

$$\begin{aligned} g^{tt} &= \frac{(\Pi - \mu r^{2-\epsilon})F + \mu r^{2-\epsilon}}{(\Pi - \mu r^{2-\epsilon})F}, \\ g^{t\phi_i} &= \frac{\mu r^{2-\epsilon}}{(\Pi - \mu r^{2-\epsilon})F} \cdot \frac{a_i}{r^2 + a_i^2}, \\ g^{\phi_i \phi_j} &= -\frac{1}{\mu_i^2 (r^2 + a_i^2)} \delta_{ij} + \frac{\mu r^{2-\epsilon}}{(\Pi - \mu r^{2-\epsilon})F} \cdot \frac{a_i a_j}{(r^2 + a_i^2)(r^2 + a_j^2)}, \\ g^{rr} &= -\frac{\Pi - \mu r^{2-\epsilon}}{\Pi F}, \\ g^{\mu_i \mu_j} &= -\frac{1}{r^2 + a_i^2} \delta_{ij} + \frac{r^2}{F} \cdot \frac{\mu_i \mu_j}{(r^2 + a_i^2)(r^2 + a_j^2)}, \end{aligned} \quad (2.5)$$

and the determinant by

$$\sqrt{|g|} = \frac{\Pi F}{r^{1-\epsilon}} \sqrt{\gamma_{D-2}}, \quad (2.6)$$

where $\sqrt{\gamma_{D-2}}$ is the determinant of the metric of S^{D-2} ,

$$\sqrt{\gamma_{D-2}} = \begin{cases} \prod_{i=1}^{n-1} \mu_i & \text{for } D = 2n + 1, \\ \frac{1}{|\alpha|} \prod_{i=1}^n \mu_i & \text{for } D = 2n + 2. \end{cases} \quad (2.7)$$

We consider a scalar field φ in the Myers-Perry black hole background. The action is

$$S = \frac{1}{2} \int d^D x \sqrt{|g|} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + S_{int}, \quad (2.8)$$

where S_{int} includes a mass term and interaction terms. Near the outer horizon, which is located at $r = r_H$ the largest root of $\Pi - \mu r^{2-\epsilon} = 0$, the kinetic term gives a dominant contribution to the action and thus we can ignore a mass and interaction terms S_{int} . Hence the action becomes near the outer horizon as

$$S = -\frac{1}{2} \int dt dr d\Omega_{D-2}(\mu r) \varphi \left[\frac{\Pi}{\Pi - \mu r^{2-\epsilon}} \left(\partial_t + \sum_{i=1}^n \frac{a_i}{r^2 + a_i^2} \partial_{\phi_i} \right)^2 - \partial_r \frac{\Pi - \mu r^{2-\epsilon}}{\Pi} \partial_r \right] \varphi. \quad (2.9)$$

Note that the terms including $\partial_{\mu_i} \varphi$ in the kinetic part are also suppressed near the horizon compared to the above terms. The scalar field φ can be expanded by the spherical harmonics $Y_{m_1 \dots m_n}^R(\mu_i, \phi_i)$ on S^{D-2} , where R is a label of representations of $SO(D-1)$ and $i \partial_{\phi_i} Y_{m_1 \dots m_n}^R(\mu_i, \phi_i) = m_i Y_{m_1 \dots m_n}^R(\mu_i, \phi_i)$. Performing the expansion, $\varphi = \sum_{R, m_i} \varphi_{m_1 \dots m_n}^R(t, r) Y_{m_1 \dots m_n}^R(\mu_i, \phi_i)$, the physics near the horizon can be effectively described by an infinite collection of massless $(1+1)$ -dimensional fields with the following action,

$$S = - \int dt dr (\mu r) (\varphi_{m_1 \dots m_n}^R)^* \left[\frac{\Pi}{\Pi - \mu r^{2-\epsilon}} \left(\partial_t + \sum_{i=1}^n \frac{i m_i a_i}{r^2 + a_i^2} \right)^2 - \partial_r \frac{\Pi - \mu r^{2-\epsilon}}{\Pi} \partial_r \right] \varphi_{m_1 \dots m_n}^R. \quad (2.10)$$

From this action we find that $\varphi_{m_1 \dots m_n}^R$ can be considered as a $(1+1)$ -dimensional complex scalar field in the background of the dilaton Φ , metric $g_{\mu\nu}$ and $U(1)$ gauge fields $A_\mu^{(\phi_i)}$,

$$\begin{aligned} \Phi = \mu r, \quad g_{tt} = \frac{\Pi - \mu r^{2-\epsilon}}{\Pi} \equiv f(r), \quad g_{rr} = -\frac{1}{f(r)}, \quad g_{tr} = 0, \\ A_t^{(\phi_i)} = -\frac{a_i}{r^2 + a_i^2}, \quad A_r^{(\phi_i)} = 0. \end{aligned} \quad (2.11)$$

The partial wave $\varphi_{m_1 \dots m_n}^R$ has the $U(1)$ charge m_i for each gauge field $A_\mu^{(\phi_i)}$.

3. Symmetries and conservation laws

When we derive Hawking radiations from anomalies at the horizon in the next section, we need to use various Ward-Takahashi identities in the two-dimensional effective theories. In this section, we derive them by reinterpreting the original conservation laws associated with general coordinate transformations in $D = 2n + 1 + \epsilon$ dimensions.

Following the general procedure in the Kaluza-Klein compactification, we write the D -dimensional metric g_{AB} ($A, B = t, r, \mu_i, \phi_i$) for the Meyers-Perry black holes in terms of a d -dimensional metric ($d = n + 1 + \epsilon$), $U(1)^n$ gauge fields and dilatons,

$$(g_{AB}) = \begin{pmatrix} g_{\alpha\beta} + h_{\phi_i \phi_j} A_\alpha^{(\phi_i)} A_\beta^{(\phi_j)} & h_{\phi_j \phi_k} A_\alpha^{(\phi_k)} \\ h_{\phi_i \phi_k} A_\beta^{(\phi_k)} & h_{\phi_i \phi_j} \end{pmatrix}, \quad (3.1)$$

where the indices α and β denote d -dimensional coordinates (t, r, μ_i) and ϕ_i 's are the angular coordinates. Note that since the metric g_{AB} does not depend on these angular coordinates there are $U(1)_{\phi_i}$ isometries. Then the D -dimensional general coordinate transformation generated by a Killing vector $\xi^{\phi_i}(t, r, \mu_i)$ becomes $U(1)_{(\phi_i)}$ gauge transformation in the d dimensions. By using the Ward-Takahashi identity with respect to this transformation, we can obtain a conservation law for $U(1)$ gauge current. Under this transformation, the fields change as

$$\delta A_{\alpha}^{(\phi_i)} = \partial_{\alpha} \xi^{\phi_i}, \quad \delta g_{\alpha\beta} = \delta h_{\phi_i\phi_j} = 0, \quad (3.2)$$

and the partition function in the above background changes as

$$\begin{aligned} \int d^D x \delta g_{AB} \frac{\delta}{\delta g_{AB}} \ln Z[g] &= i \int d^D x \xi^{\phi_i} \partial_{\alpha} \left(\sqrt{|g|} T_{\phi_i}^{\alpha} \right) \\ &= i \int d^D x \xi^{\phi_i} \left[\partial_r \left(\sqrt{|g|} T_{\phi_i}^r \right) + \sum_{i=j}^{n-1+\epsilon} \partial_{\mu_j} \left(\sqrt{|g|} T_{\phi_i}^{\mu_j} \right) \right] = 0. \end{aligned} \quad (3.3)$$

Here $T_{AB} = -\frac{i}{\sqrt{|g|}} \frac{\delta}{\delta g^{AB}} \ln Z[g]$ is a D -dimensional energy-momentum tensor and depends only on r and μ_i due to the isometries of the background. We define an r -component of a $U(1)$ gauge current in the two-dimensional field theory as

$$J_{(\phi_i)}^r(r) \equiv - \int d\Omega_{D-2} \left(\frac{\Pi F}{r^{1-\epsilon}} \right) T_{\phi_i}^r. \quad (3.4)$$

We assume that on the right hand side in eq. (3.3), the second term is negligibly small compared to the first term near the horizon of the MP black hole. This corresponds to the fact that the terms including $\partial_{\mu_i} \varphi$ could be dropped in the discussion of the effective action near the horizon in the previous section. We then can see from eq. (3.3) that the following conservation law holds near the horizon,

$$\partial_r J_{(\phi_i)}^r = 0. \quad (3.5)$$

Namely, by studying the behavior of the two-dimensional $U(1)$ gauge current, we can see the flux of the D -dimensional angular momentum.

Next, by considering a D -dimensional general coordinate transformation generated by a Killing vector $\xi^t(t, r, \mu_i)$, we can obtain another conservation law. Variations of the fields under it are

$$\begin{aligned} \delta g_{\alpha\beta} &= g_{\beta t} \nabla_{\alpha} \xi^t + g_{\alpha t} \nabla_{\beta} \xi^t, & \delta A_{\alpha}^{(\phi_j)} &= \xi^t \nabla_t A_{\alpha}^{(\phi_j)} + A_t^{(\phi_j)} \nabla_{\alpha} \xi^t, \\ \delta h_{\phi_i\phi_j} &= \xi^t \partial_t h_{\phi_i\phi_j} = 0, \end{aligned} \quad (3.6)$$

where ∇_α is the covariant derivative associated with the d -dimensional metric $g_{\alpha\beta}$. Then a change of the partition function is given by

$$\begin{aligned} & \int d^D x \delta g_{AB} \frac{\delta}{\delta g_{AB}} \ln Z[g] \\ &= i \int d^D x \xi^t \left[\partial_\alpha \left(\sqrt{|g|} T_t^\alpha \right) - \sqrt{|g|} \sum_{i=1}^n F_{t\alpha}^{(\phi_i)} T_{\phi_i}^\alpha \right] \\ &= i \int d^D x \xi^t \left[\partial_r \left(\sqrt{|g|} T_t^r \right) + \partial_{\mu_i} \left(\sqrt{|g|} T_t^{\mu_i} \right) - \sqrt{|g|} \sum_{i=1}^n F_{t\alpha}^{(\phi_i)} T_{\phi_i}^\alpha \right] = 0. \end{aligned} \quad (3.7)$$

$F_{t\alpha}^{(\phi_i)}$ are defined by the components of the field strengths of the $U(1)$ gauge fields $A_\alpha^{(\phi_i)}$,

$$F_{t\alpha}^{(\phi_i)} = \partial_t A_\alpha^{(\phi_i)} - \partial_\alpha A_t^{(\phi_i)}, \quad (3.8)$$

$$A_t^{(\phi_i)} = -\frac{\mu r^{2-\epsilon}}{(\Pi - \mu r^{2-\epsilon})F + \mu r^{2-\epsilon}} \cdot \frac{a_i}{r^2 + a_i^2}, \quad (3.9)$$

$$A_{\mu_j}^{(\phi_i)} = A_r^{(\phi_i)} = 0. \quad (3.10)$$

The equation (3.7) holds in the whole region outside the horizon but it can be simplified near the horizon as follows. Near the horizon, because of $\Pi - \mu r^{2-\epsilon} = 0$, $F_{t\mu_j}^{(\phi_i)}$ vanishes and $F_{tr}^{(\phi_i)} = -\partial_r A_t^{(\phi_i)}$ where the potential becomes

$$A_t^{(\phi_i)} \longrightarrow -\frac{a_i}{r^2 + a_i^2}, \quad (r \longrightarrow r_H). \quad (3.11)$$

These gauge field backgrounds, of course, coincide with those in eq. (2.11) derived by dimensional reduction of the action near the horizon. Furthermore we assume that in eq. (3.7), $\partial_{\mu_i} \left(\sqrt{|g|} T_t^{\mu_i} \right)$ is negligible compared to the other terms near the horizon, by a similar reason to the one described below eq. (3.4). Defining a component $T_{t(2)}^r(r)$ of the two-dimensional energy-momentum tensor as

$$T_{t(2)}^r(r) \equiv \int d\Omega_{D-2} \left(\frac{\Pi F}{r^{1-\epsilon}} \right) T_t^r, \quad (3.12)$$

we find from eq. (3.7) the following conservation equation near the horizon,

$$\partial_r T_{t(2)}^r - \sum_{i=1}^n F_{rt}^{(\phi_i)} J_{(\phi_i)}^r = 0. \quad (3.13)$$

The energy flux in D dimensions is given by the two-dimensional energy current. The second term in the above Ward-Takahashi identity can be interpreted as the dissipation of energy due to interactions of the current $J_{(\phi_i)}^r$ with the background electric field $F_{rt}^{(\phi_i)}$.

4. Quantum anomalies and Hawking fluxes

Ingoing modes near the horizon are classically irrelevant to physics outside the horizon. If we neglect the ingoing modes, in the two-dimensional effective theory near the horizon

gauge and diffeomorphism invariance are broken by quantum anomalies. The underlying theory is, of course, invariant. Therefore these anomalies are cancelled by quantum effects of ingoing modes which are irrelevant classically. In the following we show that the condition for vanishing of anomalies at the horizon leads to the correct Hawking fluxes of angular momenta and energy.

First we consider the $U(1)$ currents $J_{(\phi_i)}^r$ in the effective theory which is defined in $r \in [r_H, \infty]$. Each current corresponds to angular momentum in the ϕ^i -direction in the D -dimensional theory, as shown in the previous section. We divide the region outside the horizon into two regions, $r \in [r_H, r_H + \epsilon]$ near the horizon and $r > r_H + \epsilon$. If we omit the ingoing modes near the horizon, $U(1)$ current has gauge anomaly there. The consistent form of the abelian gauge anomaly [10–12] in two dimensions is given by

$$\nabla_\mu J_{(\phi_i)}^\mu = -\frac{m_i}{4\pi\sqrt{-g(2)}}\epsilon^{\mu\nu}\partial_\mu\mathcal{A}_\nu, \tag{4.1}$$

where $\epsilon^{01} = +1$ and μ, ν run t and r . In our case, \mathcal{A}_μ is a sum of n $U(1)$ gauge fields multiplied by charges,

$$\mathcal{A}_\mu = \sum_{j=1}^n m_j A_\mu^{(\phi_j)}, \tag{4.2}$$

$$\mathcal{A}_t = -\sum_{j=1}^n \frac{m_j a_j}{r^2 + a_j^2}, \quad \mathcal{A}_r = 0. \tag{4.3}$$

It is noted that the charge of the partial mode in the effective theory is m_i . The consistent anomaly satisfies the Wess-Zumino condition. The consistent current is derived from the quantum effective action and is not gauge covariant. We can define covariant current as

$$\tilde{J}_{(\phi_i)}^\mu \equiv J_{(\phi_i)}^\mu - \frac{m_i}{4\pi\sqrt{-g(2)}}\epsilon^{\mu\nu}\mathcal{A}_\nu, \tag{4.4}$$

which satisfies

$$\nabla_\mu \tilde{J}_{(\phi_i)}^\mu = -\frac{m_i}{4\pi\sqrt{-g(2)}}\epsilon^{\mu\nu}\mathcal{F}_{\mu\nu}. \tag{4.5}$$

Here $\mathcal{F}_{\mu\nu}$ is given by

$$\mathcal{F}_{\mu\nu} = \partial_\mu\mathcal{A}_\nu - \partial_\nu\mathcal{A}_\mu = \sum_{i=j}^n m_j F_{\mu\nu}^{(\phi_j)}. \tag{4.6}$$

In the case we consider here, the consistent current $J_{(\phi_i),H}^\mu$ in $r \in [r_H, r_H + \epsilon]$ depends only on r and satisfies the following anomalous equation with the gauge field background (2.11),

$$\partial_r J_{(\phi_i),H}^r = \frac{m_i}{4\pi}\partial_r\mathcal{A}_t. \tag{4.7}$$

In the outside region $r > r_H + \epsilon$, the current is conserved, $\partial_r J_{(\phi_i),O}^r = 0$. Hence we can solve them in each region as

$$J_{(\phi_i),O}^r = c_O^i, \tag{4.8}$$

$$J_{(\phi_i),H}^r = c_H^i + \frac{m_i}{4\pi}(\mathcal{A}_t(r) - \mathcal{A}_t(r_H)), \tag{4.9}$$

where c_O^i and c_H^i are integration constants. c_O^i is the value of the current at $r \rightarrow \infty$ and c_H^i is the value of the consistent current of the outgoing modes at the horizon $r = r_H$. The current is written as a sum in two regions

$$J_{(\phi_i)}^\mu = J_{(\phi_i),O}^\mu \Theta_+(r) + J_{(\phi_i),H}^\mu H(r), \quad (4.10)$$

where $\Theta_+(r) = \Theta(r - r_H - \epsilon)$ and $H(r) = 1 - \Theta_+(r)$ are step functions which are defined in the region $r \geq r_H$. Note that this current is a part of the total current because we omitted the ingoing modes near the horizon. The total current including a contribution of the ingoing modes is given by

$$J_{(\phi_i)total}^\mu = J_{(\phi_i)}^\mu + K_{(\phi_i)}^\mu, \quad (4.11)$$

where

$$K_{(\phi_i)}^r = -\frac{m_i}{4\pi} \mathcal{A}_t(r) H(r). \quad (4.12)$$

This contribution cancels the anomalous part of $J_{(\phi_i)}^\mu$ near the horizon.

In the previous papers [1–3], the invariance of the quantum effective action under gauge and general coordinate transformations was considered. Requiring the invariance can determine the relation between the integration constants c_O^i and c_H^i . This method has revealed the significance of each contribution from the ingoing and outgoing modes, but the relation itself can be easily obtained by imposing the conservation of the total current, $\partial_r J_{(\phi_i)total}^r = 0$. This condition gives the relation,

$$c_O^i = c_H^i - \frac{m_i}{4\pi} \mathcal{A}_t(r_H). \quad (4.13)$$

In order to fix the value of the current, we need a further boundary condition at the horizon. Here we impose that the covariant form of the outgoing current should vanish at the horizon. The outgoing modes can produce possible divergences for physical quantities seen by infalling observers into black holes, as discussed in, for example, [14, 6]. The physical vacuum in the black hole backgrounds should be chosen as such that these unphysical divergences should vanish. In our case, this condition corresponds to the above boundary condition. We discuss it in the appendix again. Another condition to specify the vacuum is the boundary condition for the ingoing currents $K_{(\phi_i)}^\mu$. In eq. (4.12) we have already chosen an integration constant such that it vanishes at $r \rightarrow \infty$. This condition corresponds to taking Unruh vacuum, in stead of Hartle-Hawking vacuum.

Since the covariant current is given by $\tilde{J}_{(\phi_i)}^r = J_{(\phi_i)}^r + \frac{m_i}{4\pi} \mathcal{A}_t(r) H(r)$, the condition $\tilde{J}_{(\phi_i)}^r(r_H) = 0$ determines the value of the charge flux at $r \rightarrow \infty$ as

$$c_O^i = -\frac{m_i}{2\pi} \mathcal{A}_t(r_H) = \frac{m_i}{2\pi} \sum_{j=1}^n \frac{m_j a_j}{r_H^2 + a_j^2}. \quad (4.14)$$

This coincides with the flux of angular momentum associated with the Hawking radiation.

Next we consider the flux of energy-momentum radiated from the Myers-Perry black holes. Omitting the ingoing modes in the near horizon region $r_H \leq r \leq r_H + \epsilon$, the

consistent energy-momentum tensor $T_{\nu,H}^\mu$ satisfies a modified conservation equation with the consistent anomalies. In two dimensions the energy-momentum tensor for right-hand modes satisfies the following Ward-Takahashi identity with $U(1)$ gauge fields $A_\mu^{(\phi_i)}$ and dilaton Φ backgrounds,

$$\nabla_\mu T_\nu^\mu = \sum_{i=1}^n \left(F_{\mu\nu}^{(\phi_i)} J_{(\phi_i)}^\mu + A_\nu^{(\phi_i)} \nabla_\mu J_{(\phi_i)}^\mu \right) - \frac{\partial_\nu \Phi}{\sqrt{-g(2)}} \frac{\delta S}{\delta \Phi} + \mathcal{A}_\nu, \quad (4.15)$$

where \mathcal{A}_μ is the consistent gravitational anomaly [13] which is given by

$$\mathcal{A}_\mu = \frac{1}{96\pi\sqrt{-g(2)}} \epsilon^{\nu\rho} \partial_\rho \partial_\sigma \Gamma_{\mu\nu}^\sigma. \quad (4.16)$$

This energy-momentum tensor is not covariant under general coordinate transformations. On the other hand the covariant energy-momentum \tilde{T}_ν^μ satisfies the Ward-Takahashi identity with the same form as the above but the anomaly term \mathcal{A}_μ is replaced with the covariant one $\tilde{\mathcal{A}}_\mu = -\frac{1}{96\pi\sqrt{-g(2)}} \epsilon_{\mu\nu} \nabla^\nu R$.

In the case considered here, the $\nu = t$ component of the Ward-Takahashi identity in the consistent form becomes

$$\partial_r T_{t,H}^r = \sum_{i=1}^n \left(F_{rt}^{(\phi_i)} J_{(\phi_i)}^r + A_t^{(\phi_i)} \nabla_\mu J_{(\phi_i)}^\mu \right) + \partial_r N_t^r(r), \quad (4.17)$$

where background fields (2.11) are used. $N_t^r(r)$ is defined by $\mathcal{A}_t = \partial_r N_t^r$,

$$N_t^r(r) = \frac{1}{192\pi} (f'^2(r) + f(r)f''(r)). \quad (4.18)$$

Eq. (4.17) is the same Ward-Takahashi identity as eq. (3.13) if there are no anomalous terms. See [2] for the Ward-Takahashi identity in presence of anomalies. In eq. (4.17), the first and second terms in the right hand side are combined in terms of the covariant current $\tilde{J}_{(\phi_i)}^r$ as $F_{rt}^{(\phi_i)} \tilde{J}_{(\phi_i)}^r$. By substituting $\tilde{J}_{(\phi_i)}^r = c_O^i + \frac{m_i}{2\pi} \mathcal{A}_t(r)$ into the equation, $T_{t,H}^r$ is obtained as

$$T_{t,H}^r = a_H + \int_{r_H}^r dr \partial_r \left(-\frac{1}{2\pi} \mathcal{A}_t(r_H) \mathcal{A}_t(r) + \frac{1}{4\pi} \mathcal{A}_t^2(r) + N_t^r(r) \right). \quad (4.19)$$

On the other hand, the energy-momentum tensor $T_{t,O}^r$ in the outside region $r > r_H$ satisfies

$$\partial_r T_{t,O}^r = \sum_{i=1}^n F_{rt}^{(\phi_i)} J_{(\phi_i),O}^r. \quad (4.20)$$

By using $J_{(\phi_i),O}^r = c_O^i$ this is solved as

$$T_{t,O}^r = a_O - \frac{1}{2\pi} \mathcal{A}_t(r_H) \mathcal{A}_t(r). \quad (4.21)$$

The energy-momentum tensor combines contributions from these two regions, $T_\nu^\mu = T_{\nu,O}^\mu \Theta_+ + T_{\nu,H}^\mu H$. This does not contain a contribution from the ingoing modes near the horizon. The total energy-momentum tensor is a sum of T_ν^μ and U_ν^μ , where

$$U_t^r = - \left(\frac{1}{4\pi} \mathcal{A}_t^2(r) + N_t^r(r) \right) H(r) \quad (4.22)$$

is a contribution from the ingoing modes. To determine a constant part of U_t^r , we require the condition that the current should vanish at $r \rightarrow \infty$. This condition corresponds to vanishing ingoing energy flow at $r \rightarrow \infty$.

We can again obtain a relation between the integration constants a_O and a_H from the conservation of total energy-momentum tensor

$$a_O = a_H + \frac{1}{4\pi} \mathcal{A}_t^2(r_H) - N_t^r(r_H). \tag{4.23}$$

In order to determine the flux of energy a_O at $r \rightarrow \infty$, we impose a vanishing condition for the covariant energy-momentum tensor at the horizon, $\tilde{T}_t^r(r_H) = 0$. This corresponds to the regularity condition for the energy-momentum tensor at the future horizon. In this case, the covariant anomaly is given by $\tilde{\mathcal{A}}_t = \partial_r \tilde{N}_t^r$ where

$$\tilde{N}_t^r = \frac{1}{96\pi} \left(f f'' - \frac{1}{2} (f')^2 \right), \tag{4.24}$$

and the covariant energy-momentum tensor is related to the consistent one as

$$\tilde{T}_t^r = T_t^r + \frac{1}{192\pi} (f f'' - 2(f')^2). \tag{4.25}$$

Therefore a_H is determined as

$$a_H = \frac{\kappa^2}{24\pi} = 2N_t^r(r_H), \tag{4.26}$$

where the surface gravity κ at the horizon is

$$\kappa = \frac{2\pi}{\beta} = \frac{1}{2} f'(r_H) = \left. \frac{\Pi'(r) - (2 - \epsilon)\mu r^{1-\epsilon}}{2\mu r^{2-\epsilon}} \right|_{r=r_H}. \tag{4.27}$$

The flux of energy-momentum tensor is given by

$$\begin{aligned} a_O &= \frac{1}{4\pi} \mathcal{A}_t^2(r_H) + N_t^r(r_H) \\ &= \frac{1}{4\pi} \left(\sum_{i=1}^n \frac{m_i a_i}{r_H^2 + a_i^2} \right)^2 + \frac{\pi}{12\beta^2}. \end{aligned} \tag{4.28}$$

Note that the flux is independent of the dimension D of the space-time. In the non-rotating cases ($a_i = 0$), the Hawking flux is proportional to $(T_H)^2$ where $T_H = 1/\beta$ is the Hawking temperature of the black hole. In D dimensions the energy density of the black body radiation with temperature T_H is proportional to $(T_H)^D$ (Stephan-Boltzmann law.) Since the area of black holes is proportional to $\mathcal{A} \sim M^{D-2} \sim (T_H)^{2-D}$, the total flux behaves as $(T_H)^2$, which is like the two-dimensional Stephan-Boltzmann law.

5. Discussions

In this paper we have applied the method of quantum anomalies to derive the Hawking flux from Myers-Perry black holes with multiple angular momenta in various dimensions.

The method adopted here made only use of the quantum anomalies at the horizon and in this sense it is very universal. Namely, it does not depend on the details of the quantum fields away from the horizon. But we have obtained only the total flux of energy or charges and a natural question is whether we can obtain more detailed information about the black body radiation from black holes.

Black body spectrum of the Hawking radiations with the Hawking temperature is deformed by the gray body factor due to the effect of scatterings away from the horizon. This is, of course, not universal and we need more information about the black hole background away from the horizon. But the radiation before it is modified may be universally given and there is a chance that we can obtain the black body spectrum based on the quantum anomalies. Near the horizon, quantum fields behave as an infinite set of two-dimensional conformal fields. Such fields have infinitely many conserved currents with higher spins in addition to the energy momentum tensor or gauge currents. In curved space-times, they will acquire quantum anomalies if the fields are chiral, and values of the anomalies for these higher spin currents can determine the black body spectrum of the Hawking radiations. We would like to report it in our future publication [15].

Another issue is the entropy of black holes. Although the black hole entropy is well understood macroscopically, its microscopic understanding is yet incomplete. Since black hole entropy is also universally determined as the Hawking radiation, we may expect that it can be given only by some universal quantum effects at the horizon. The black hole entropy may be given by the number of degrees of freedom of the ingoing modes of some gravitational modes at the horizon (which are classically irrelevant but quantum mechanically important to physics outside the black holes). We calculated the entropy based on the idea and the result is proportional to the area, but the coefficient is slightly different from the Bekenstein-Hawking entropy. Since the entropy must be universal as well as the Hawking radiation, we believe that our approach for the quantum physics near the horizon will be important to investigate the thermodynamic properties of black holes. We would like to further study and report it in near future.

Note added. After completing this work we found a preprint [16] in which Hawking radiations in general Kerr-(anti)de Sitter black holes are studied.

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A. What is different between [1] and [2, 3]

Since there seem to be some confusions among the readers of [1] and [2, 3], we will explain the difference between them here. Although the basic ideas are essentially the same, there are the following differences in the calculations.

In the paper [1], *outgoing modes* near the horizon are eliminated and thus effective theory is chiral there. Then effective action for the metric due to matter fields becomes anomalous near the horizon with respect to general coordinate transformations. The Hawking

flux of energy-momentum tensor is determined so that it cancels the gravitational anomaly in the *consistent* form at the horizon.

On the other hand, in [2, 3] and the present paper, *ingoing modes*, which are classically irrelevant to physics outside the horizon, are integrated out near the horizon. The Hawking fluxes are determined by the requirement that the *covariant* current or energy-momentum tensor should vanish at the horizon, instead of the consistent current.

In the case of the Schwarzschild black hole, both of them give the same answer. This is because the change of the sign for the coefficients of anomalies due to whether we consider outgoing or ingoing modes can be cancelled by the change of sign whether we consider the consistent or covariant form of gravitational anomalies at the horizon. (See the coefficients of $(f')^2$ terms in eqs. (4.18) and (4.24).)

In more general cases of charged or rotating black holes, however, we have to consider gauge currents in addition to energy flow and hence both of the gravitational and gauge anomalies must be considered. Since the values of gauge anomalies at the horizon differ by a factor 2, not by their signs, between the consistent and the covariant currents (eqs. (4.1) and (4.5)), it cannot cancel the first change of the sign. Because of it, when we applied the calculation [1] to charged or rotating black holes, we could not reproduce the correct value of the Hawking fluxes.

In the appendix in [3], we have calculated the effective action for two-dimensional free fields in charged black hole backgrounds and obtained the form of gauge currents and energy-momentum tensor directly imposing the regularity condition that fluxes of current or energy-momentum tensor seen by a freely falling observer should not be singular at the horizon;

$$J^r = -\frac{e^2}{2\pi}A_t(r_H), \tag{A.1}$$

$$T_t^r = -\frac{e^2}{2\pi}A_t(r_H)A_t(r) + \frac{1}{192\pi}f'^2(r_H) + \frac{e^2}{4\pi}A_t^2(r_H). \tag{A.2}$$

In the cases of rotating black holes investigated in this paper, the above current and background $U(1)$ gauge field should be regarded as those for each $U(1)$ gauge symmetry corresponding to the diffeomorphism in the ϕ_i -direction. Then it is easily found that these are equivalent to the current $J_{(\phi_i),O}^r$ and energy-momentum tensor $T_{t,O}^r$ which are determined imposing the condition that the covariant current should vanish at the horizon, not the consistent current.

References

- [1] S.P. Robinson and F. Wilczek, *A relationship between Hawking radiation and gravitational anomalies*, *Phys. Rev. Lett.* **95** (2005) 011303 [[gr-qc/0502074](#)].
- [2] S. Iso, H. Umetsu and F. Wilczek, *Hawking radiation from charged black holes via gauge and gravitational anomalies*, *Phys. Rev. Lett.* **96** (2006) 151302 [[hep-th/0602146](#)].
- [3] S. Iso, H. Umetsu and F. Wilczek, *Anomalies, Hawking radiations and regularity in rotating black holes*, *Phys. Rev.* **D 74** (2006) 044017 [[hep-th/0606018](#)].

- [4] S.W. Hawking, *Particle creation by black holes*, *Commun. Math. Phys.* **43** (1975) 199; *Black hole explosions*, *Nature (London)* **248** (1974) 30.
- [5] J.B. Hartle and S.W. Hawking, *Path integral derivation of black hole radiance*, *Phys. Rev. D* **13** (1976) 2188.
- [6] S.M. Christensen and S.A. Fulling, *Trace anomalies and the Hawking effect*, *Phys. Rev. D* **15** (1977) 2088.
- [7] K. Murata and J. Soda, *Hawking radiation from rotating black holes and gravitational anomalies*, *Phys. Rev. D* **74** (2006) 044018 [[hep-th/0606069](#)].
- [8] E.C. Vagenas and S. Das, *Gravitational anomalies, Hawking radiation and spherically symmetric black holes*, *JHEP* **10** (2006) 025 [[hep-th/0606077](#)];
M.R. Setare, *Gauge and gravitational anomalies and Hawking radiation of rotating btz black holes*, *Eur. Phys. J. C* **49** (2007) 865 [[hep-th/0608080](#)].
- [9] R.C. Myers and M.J. Perry, *Black holes in higher dimensional space-times*, *Ann. Phys. (NY)* **172** (1986) 304.
- [10] W.A. Bardeen and B. Zumino, *Consistent and covariant anomalies in gauge and gravitational theories*, *Nucl. Phys. B* **244** (1984) 421.
- [11] R. Bertlmann, *Anomalies in quantum field theory* International Series of Monographs on Physics **91** Oxford Science Publications, Oxford, 2000.
- [12] K. Fujikawa and H. Suzuki, *Path integrals and quantum anomalies*, Oxford Science Publications, Oxford, 2004.
- [13] L. Alvarez-Gaumé and E. Witten, *Gravitational anomalies*, *Nucl. Phys. B* **234** (1984) 269.
- [14] W.G. Unruh, *Notes on black hole evaporation*, *Phys. Rev. D* **14** (1976) 870.
- [15] S. Iso, T. Morita and H. Umetsu, to appear.
- [16] Z. Xu and B. Chen, *Hawking radiation from general kerr-(anti)de Sitter black holes*, *Phys. Rev. D* **75** (2007) 024041 [[hep-th/0612261](#)].